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# SEMESTER ONE

# MATHEMATICS SPECIALIST

# REVISION 1

# UNIT 3

# 2016

# SOLUTIONS

# Section One

1. (10 marks)

(a)

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & -1 & 1 & 6 \\ 1 & 0 & 1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 2 & 0 & -4 \\ 0 & 1 & 0 & -2 \end{bmatrix} \quad R_1 - R_2 \quad ✓ \quad R_1 - R_3 \quad ✓$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 0 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2 - 2R_3 \quad ✓$$

$0 = 0$

∴ There are an infinite number of solutions.

None of the planes are parallel.

There are either two or three identical planes or the three planes intersect in a common line.

None of the planes have identical/equivalent equations so the three planes meet in a common line. ✓ (4)

(b) Two of the planes are parallel. Therefore there is no intersection. ✓

$x + y + z = 2$ and $-x - y - z = 1 \Leftrightarrow x + y + z = -1$ are parallel ✓ (2)

(c)

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 11 \\ 0 & 4 & 4 & 20 \end{bmatrix} \quad 2R_1 - R_2 \quad 3R_1 - R_3 \quad ✓ \quad ✓$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 11 \\ 0 & 1 & 1 & 5 \end{bmatrix} \quad R_3 \div 4$$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & 2 & 6 \end{bmatrix} \quad R_2 - R_3 \quad ✓$$

$z = 3$

$y + 9 = 11 \rightarrow y = 2$

$x + 2 + 3 = 6 \rightarrow x = 1$

The point of intersection is (1, 2, 3) ✓ (4)

2. (6 marks)

(a) $f(x) = \sin(|x|)$ ✓✓ (2)

(b) $g(x) = |x^3 + 1|$ ✓✓ (2)

(c) $p(x) = e^{x-1}$ ✓
$q(x) = 1 + \ln(x)$ ✓ (2)

3. (13 marks)

(a) $z^3 - z^2 - 4 = 0$

Let $P(z) = z^3 - z^2 - 4$

$P(2) = 8 - 4 - 4 = 0$

$\therefore z = 2$ so $z - 2$ is a factor ✓

$$\begin{array}{r} z^2 + z + 2 \\ z-2\overline{)z^3 - z^2 + 0z - 4} \\ -\underline{(z^3 - 2z^2)} \\ z^2 + 0z \\ -\underline{(z^2 - 2z)} \\ 2z - 4 \\ -\underline{(2z - 4)} \\ 0 \end{array}$$

$z = 2$ or $z^2 + z + 2 = 0$ ✓

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1-8}}{2} \qquad \Delta = -7 = 7i^2$$

$$z = \frac{-1 \pm i\sqrt{7}}{2}$$

$$\therefore \ z = \frac{-1 \pm i\sqrt{7}}{2} \text{ or } z = 2$$ (4)

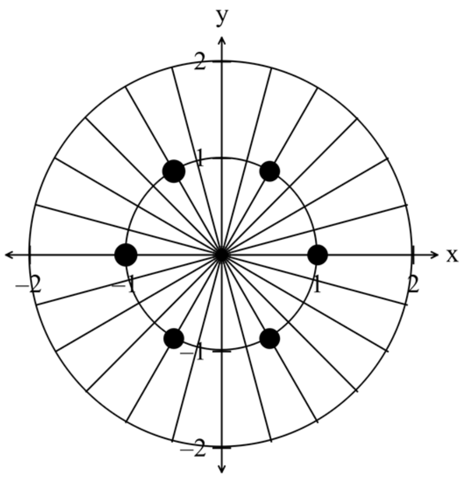
✓✓

(b) $z^3 = 8\,cis(\pi)$ ✓✓

$z^3 = -8$ ✓

(3)

(c) (i)




✓✓ -1 per error

(2)

(ii) $z^6 = 1$

$z^6 = cis(0 + 2n\pi) \quad n \in R$

$z = \left(cis(2n\pi)\right)^{\frac{1}{6}}$

$z = cis\left(\frac{2n\pi}{6}\right)$ ✓

$z = cis\left(\frac{n\pi}{3}\right)$

$n = 0, \quad z = cis(0) = 1$

$n = 1, \quad z = cis\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{i\sqrt{3}}{2}$ ✓

$n = 2, \quad z = cis\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{i\sqrt{3}}{2}$ ✓

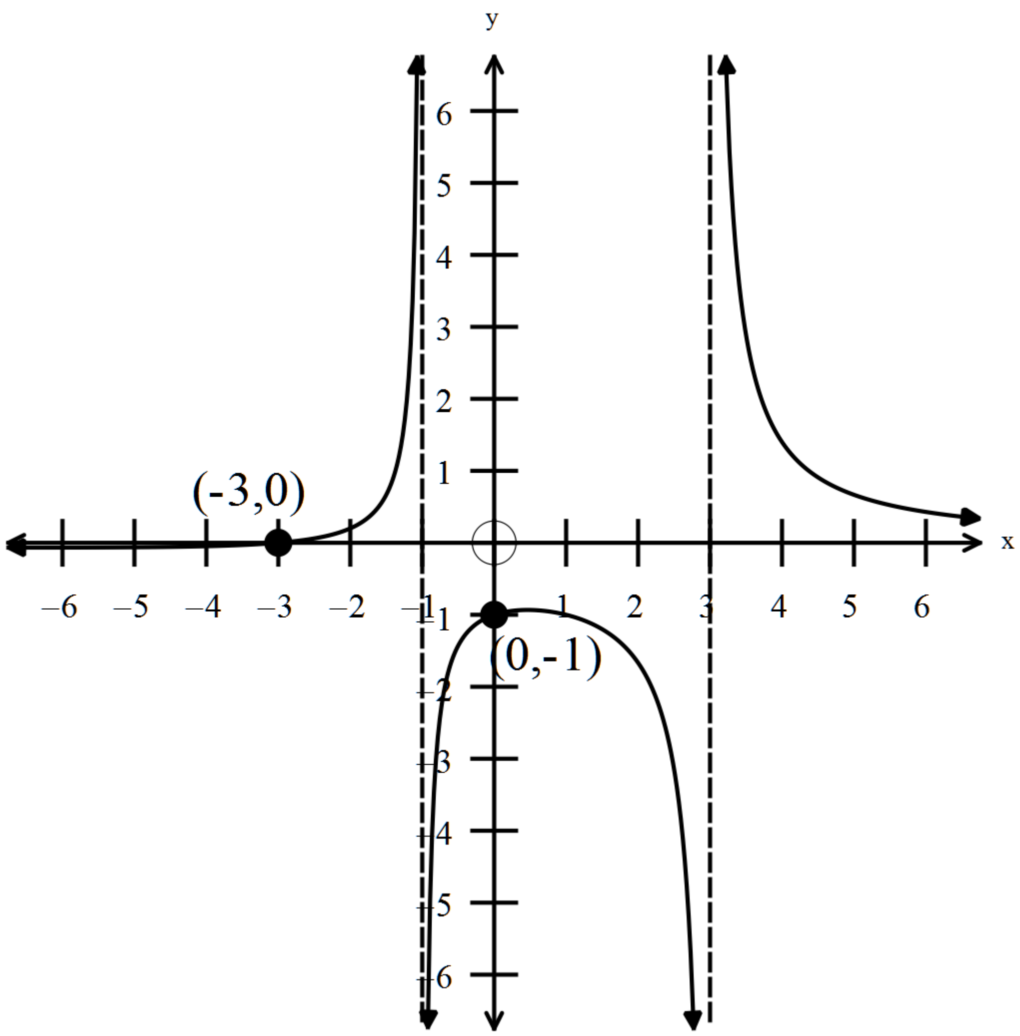
$n = 3, \quad z = cis(\pi) = -1$

$n = -1, \quad z = cis\left(-\frac{\pi}{3}\right) = \frac{1}{2} - \frac{i\sqrt{3}}{2}$ ✓

$n = -2, \quad z = cis\left(-\frac{2\pi}{3}\right) = -\frac{1}{2} - \frac{i\sqrt{3}}{2}$ -1 per error (4)

4. (8 marks)

(a) $f(x)=\dfrac{(x+3)}{(x+1)(x-3)}$



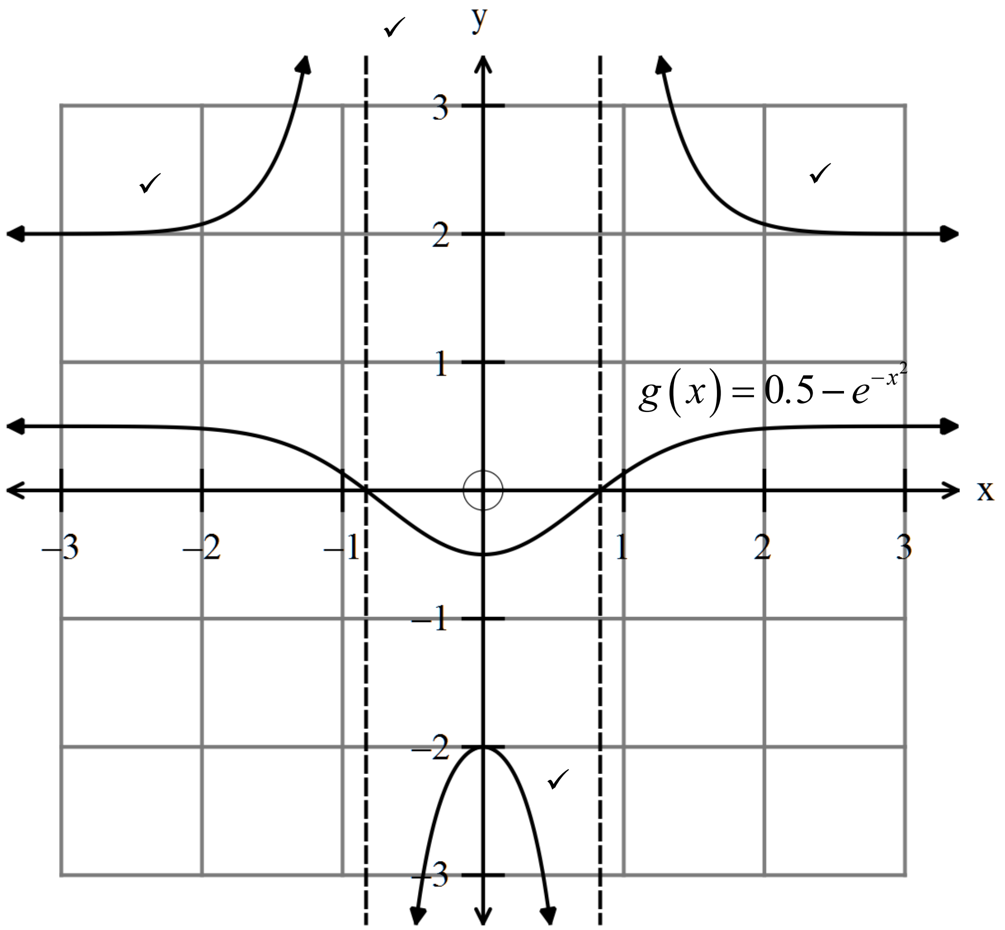

Vertical asymptotes at $x=-1,\ \ x=3$ ✓ ✓

$x$ intercept at $x=-3$ ✓

$y$ intercept at $y=-1$ ✓

General shape – maximum turning point (w/0 cutting x axis); limits as $x\to\pm\infty$; limits about asymptotes ✓ -1/error (4)

(b)




(4)

5. (13 marks)

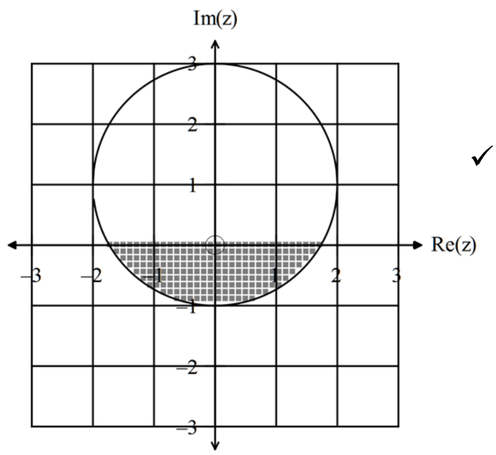
(a) $\left(3\sqrt{3}+\sqrt{2}i\right)\left(2\sqrt{3}-2\sqrt{2}i\right)$.

$=18+2i\sqrt{6}-6i\sqrt{6}-4i^2$ ✓✓

$=22-4i\sqrt{6}$ ✓

(3)

(b)




✓✓✓ -1/error

(3)

(c)

$$(1-i)^{10} = \left(\sqrt{2}\, cis\left(-\frac{\pi}{4}\right)\right)^{10}$$

$$= \left(\sqrt{2}\right)^{10} cis\left(-\frac{10\pi}{4}\right)$$

$$= 2^5\, cis\left(-\frac{5\pi}{2}\right) \quad ✓$$

$$= 32 cis\left(-\frac{\pi}{2}\right)$$

$$= 32(0-i)$$

$$(1-i)^{10} = -32i \quad ✓$$

(2)

(d) $Re\left(\dfrac{3-4i}{1+2i}\right) = Re(-1-2i) = -1$

✓ ✓ (2)

(e) (i) $z = cis\left(\dfrac{2\pi}{3}\right) = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i \Rightarrow \bar{z} = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$ ✓ (1)

(ii) $z^2 = cis\left(\dfrac{2\pi}{3}\right)^2 = cis\left(\dfrac{4\pi}{3}\right) = -\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}i$ ✓ (1)

(ii) $\dfrac{1}{z^2} = \dfrac{1}{\left(cis\left(\dfrac{2\pi}{3}\right)\right)^2} = \dfrac{1}{cis\left(\dfrac{4\pi}{3}\right)} = cis\left(-\dfrac{4\pi}{3}\right) = -\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$ ✓ (1)

## END OF SECTION ONE

# Section Two

6. (6 marks)

(a) $\boldsymbol{v}(t)=(2cos(2t))\boldsymbol{i}+(-sin(t))\boldsymbol{j}$

$\boldsymbol{r}(t)=\int((2cos(2t))\boldsymbol{i}+(-sin(t))\boldsymbol{j})dt$

$\boldsymbol{r}(t)=(sin(2t))\boldsymbol{i}+(cos(t))\boldsymbol{j}+\boldsymbol{c}$ ✓

$\boldsymbol{r}(\pi)=-\boldsymbol{j}$ so

$-\boldsymbol{j}=(sin(2\pi))\boldsymbol{i}+(cos(\pi))\boldsymbol{j}+\boldsymbol{c}$

$-\boldsymbol{j}=-\boldsymbol{j}+\boldsymbol{c} \Rightarrow \boldsymbol{c}=\boldsymbol{0}$ ✓

$\therefore \boldsymbol{r}(t)=(sin(2t))\boldsymbol{i}+(cos(t))\boldsymbol{j}$

$\boldsymbol{a}(t)=(-4sin(2t))\boldsymbol{i}+(-cos(t))\boldsymbol{j}$ ✓

(3)

(b) $\boldsymbol{r}\left(\frac{3\pi}{2}\right)=(sin(3\pi))\boldsymbol{i}+\left(cos\left(\frac{3\pi}{2}\right)\right)\boldsymbol{j}=0\boldsymbol{i}+0\boldsymbol{j}=\boldsymbol{0}$ ✓

$\boldsymbol{v}\left(\frac{3\pi}{2}\right)=(2cos(3\pi))\boldsymbol{i}+\left(-sin\left(\frac{3\pi}{2}\right)\right)\boldsymbol{j}=-2\boldsymbol{i}+\boldsymbol{j}$ ✓ (2)

(c) Show that $4\boldsymbol{r}(t)+\boldsymbol{a}(t)=3cos(t)\boldsymbol{j}$.

$4\boldsymbol{r}(t)+\boldsymbol{a}(t)=4\begin{pmatrix}sin(2t)\\cos(t)\end{pmatrix}+\begin{pmatrix}-4sin(2t)\\-cos(t)\end{pmatrix}$

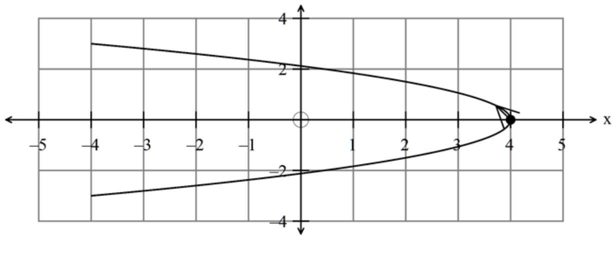
$=\begin{pmatrix}0\\3cos(t)\end{pmatrix}$ ✓ (1)

$=(3cos(t))\boldsymbol{j}$

7. (20 marks)

(a) $\boldsymbol{r}(0)=(4cos(0))\boldsymbol{i}+(3sin(0))\boldsymbol{j}=\begin{pmatrix}4\\0\end{pmatrix}$ ✓

$\boldsymbol{r}(0^+)=(4cos(0^+))\boldsymbol{i}+(3sin(0^+))\boldsymbol{j}=\begin{pmatrix}4^-\\0^+\end{pmatrix}$ ✓

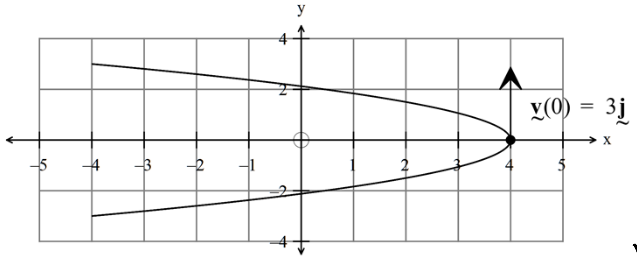



✓

(3)

(b) $\boldsymbol{v}(t) = (-8\sin(2t))\boldsymbol{i} + (3\cos(t))\boldsymbol{j}$ ✓✓

$\boldsymbol{v}(0) = 3\boldsymbol{j}$ ✓




✓

(4)

(c) $\boldsymbol{a}(t) = (-16\cos(2t))\boldsymbol{i} + (-3\sin(t))\boldsymbol{j}$ ✓✓ (2)

(d) $\boldsymbol{a}(t) = \begin{pmatrix} 16 \\ -3 \end{pmatrix} \Rightarrow t = \frac{\pi}{2}$ ✓

$\boldsymbol{r}\left(\frac{\pi}{2}\right) = (4\cos(\pi))\boldsymbol{i} + \left(3\sin\left(\frac{\pi}{2}\right)\right)\boldsymbol{j}$

$\boldsymbol{r}\left(\frac{\pi}{2}\right) = -4\boldsymbol{i} + 3\boldsymbol{j}$ ✓

(2)

(e) $\boldsymbol{a}(t) = (-16\cos(2t))\boldsymbol{i} + (-3\sin(t))\boldsymbol{j}$

$\boldsymbol{r}(t) = (4\cos(2t))\boldsymbol{i} + (3\sin(t))\boldsymbol{j}.$

$\boldsymbol{a}(t) \neq k\,\boldsymbol{r}(t)$ ✓

as $-1\times 3 = -3$ *but* $-1\times 4 \neq -16$ ✓ (2)

(f) If $\boldsymbol{a}(t) = \boldsymbol{0}$ then $\boldsymbol{a}(t) = (-16\cos(2t))\boldsymbol{i} + (-3\sin(t))\boldsymbol{j} = \boldsymbol{0}$

$x = -16\cos(2t) = 0$ and $y = (-3\sin(t)) = 0$ ✓

$2t = \frac{\pi}{2} + k\pi$ $\quad t = 0, \pi, 2\pi$ ✓

$t = \frac{\pi}{4} + \frac{k\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ ✓

✓

The x and y coordinates are never zero at the same time so $\boldsymbol{a}(t) \neq \boldsymbol{0}$ (4)

(g) $\boldsymbol{r}(t) = (4\cos(2t))\boldsymbol{i} + (3\sin(t))\boldsymbol{j}$

$x = (4\cos(2t))$ $\quad y = (3\sin(t))$ ✓

✓ $x = 4(1 - 2\sin^2(t))$ $\quad \sin(t) = \dfrac{y}{3}$

$x = 4\left(1 - \dfrac{2y^2}{9}\right)$ ✓

(3)

8. (10 marks)

(a) $A(3, 4, 0)$, $B(4, -3, 0)$ and $C(0, 0, 5)$.

$$\boldsymbol{AB} = \begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix}, \ \boldsymbol{AC} = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix} \quad ✓$$

$$\boldsymbol{r}(t) = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + s\begin{pmatrix} 1 \\ -7 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix} \quad ✓$$

There are alternative answers as they can also use ***BC*** as a direction vector and then any of the points A,B or C for the equation.

(2)

(b) $x^2 + y^2 + z^2 = 25$ ✓✓ (2)

(c) (i) $$\boldsymbol{r}_{ball}(t) = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} \quad ✓$$

Hits the ground at $2 - t = 0$ i.e. $t = 2$ seconds ✓ (2)

(ii) $$\boldsymbol{r}_{Paul}(t) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix} \quad ✓$$

$$\boldsymbol{r}_{Paul}(2) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 3.5 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \\ 0 \end{pmatrix} \quad ✓$$

$$\boldsymbol{r}_{ball}(2) = \begin{pmatrix} -3 \\ 5 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \\ 0 \end{pmatrix} \quad ✓$$

Both Paul and the ball are at the same point $Q(5,11,0)$ at ground level when $t = 2$, so Paul catches the ball. (3)

(ii) $2\times\left|\begin{pmatrix}1\\3.5\\-1\end{pmatrix}\right| = 2\times\sqrt{1+12.25+1} = 7.55$ m ✓ (1)

9. (6 marks)

(a) (i) $\boldsymbol{MN} = \begin{pmatrix}-4\\-6\\-8\end{pmatrix}$ ✓

$r(t) = \begin{pmatrix}2\\1\\3\end{pmatrix} + t\begin{pmatrix}-4\\-6\\-8\end{pmatrix}$ ✓ (2)

(ii) $x = 2-4t \qquad y = 1-6t \qquad z = 3-8t$ ✓

$t = \dfrac{2-x}{4} \qquad t = \dfrac{1-y}{6} \qquad t = \dfrac{3-z}{8}$

$so \quad \dfrac{2-x}{4} = \dfrac{1-y}{6} = \dfrac{3-z}{8}$ ✓

(2)

(b) $L_1:\ \boldsymbol{r}_1(t) = \begin{pmatrix}1\\2\\3\end{pmatrix} + t\begin{pmatrix}-1\\0\\-1\end{pmatrix} \Rightarrow \begin{matrix}x = 1-t\\ y = 2\\ z = 3-t\end{matrix}$

$L_2:\ \boldsymbol{r}_2(s) = \begin{pmatrix}0\\2\\2\end{pmatrix} + s\begin{pmatrix}0\\0\\2\end{pmatrix} \Rightarrow \begin{matrix}x = 0\\ y = 2\\ z = 2+2s\end{matrix}$

It can be seen that $y = 2$. If $x = 0 \Rightarrow t = 1 \ \ i.e. \ \ z = 2 \ \Rightarrow (0,2,2)$

If $z = 2$ then $s = 0$ which does not contradict the $x$ and $y$ values and gives $(0,2,2)$. ✓ logic

Yes, the lines intersect, and do so at $(0,2,2)$. ✓

(2)

10. (9 marks)

(a) $\boldsymbol{AB} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}, \boldsymbol{AC} = \begin{pmatrix} 2 \\ -4 \\ -6 \end{pmatrix}$ ✓

$Area_{\Delta} = \frac{1}{2} a \times b \times sin(C) \quad and \quad |\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \, sin(\theta)$

$Area_{\Delta} = \frac{1}{2}|\boldsymbol{AB} \times \boldsymbol{AC}|$ ✓

$= \frac{1}{2}\left|\begin{pmatrix} -28 \\ -2 \\ -8 \end{pmatrix}\right|$ ✓

$= \frac{1}{2}\sqrt{784 + 4 + 64}$

$Area_{\Delta} = 14.6 \; units^2$ ✓

(4)

(b) $AB : AC = |\boldsymbol{AB}| : |\boldsymbol{AC}| = \sqrt{21} : \sqrt{56} = \frac{\sqrt{7 \times 3}}{\sqrt{7 \times 8}} = \sqrt{3} : 2\sqrt{2}$ ✓ ✓ (2)

(c) $P(1.5, 3, 1) \quad Q(2, 0, 0)$ ✓

$\boldsymbol{PQ} = \begin{pmatrix} 0.5 \\ -3 \\ -1 \end{pmatrix} \quad \boldsymbol{BC} = \begin{pmatrix} 1 \\ -6 \\ -2 \end{pmatrix}$ ✓

$\boldsymbol{BC} = 2\begin{pmatrix} 0.5 \\ -3 \\ -1 \end{pmatrix}$ ✓

$\therefore \; 2\,\boldsymbol{PQ} = \boldsymbol{BC}$

Therefore $PQ$ is parallel to $BC$. (3)

11. (13 marks)

(a)
$$\frac{(xy)^3}{\sqrt{z}} = \frac{\left(cis\left(\frac{\pi}{4}\right)\right)^3 (1-i)^3}{\left(1+\sqrt{3}i\right)^{1/2}}$$

$$= \frac{\left(cis\left(\frac{3\pi}{4}\right)\right)\left(\left(\sqrt{2}\right)^3 cis\left(-\frac{3\pi}{4}\right)\right)}{\left(2cis\left(\frac{\pi}{3}\right)\right)^{1/2}}$$

$$= \frac{2\sqrt{2}\left(cis\left(\frac{3\pi}{4}\right)\right)\left(cis\left(-\frac{3\pi}{4}\right)\right)}{\left(\sqrt{2}\, cis\left(\frac{\pi}{6}\right)\right)} \quad ✓$$

$$= 2\left(cis\left(\frac{3\pi}{4} - \frac{3\pi}{4} - \frac{\pi}{6}\right)\right) \quad ✓$$

$$= 2\left(cis\left(-\frac{\pi}{6}\right)\right) \quad ✓$$

$$= 2\left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)$$

$$= \sqrt{3} - i \quad ✓$$

(4)

(b) $\left\{ z: 1 \le |z| \le 2 \ \cap \ -\frac{3\pi}{4} \le arg(z) \le \frac{\pi}{4} \right\}$ ✓ ✓ ✓ correct inequalities (3)

(c) (i) Given $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ show that $\overline{z_1 z_2} = \overline{z_1} \times \overline{z_2}$.

$$\overline{z_1} \times \overline{z_2} = \overline{(x_1 + iy_1)} \times \overline{(x_2 + iy_2)}$$
$$= (x_1 - iy_1) \times (x_2 - iy_2) \quad ✓$$
$$= x_1x_2 + i^2 y_1y_2 + i(-x_1y_2 - x_2y_1)$$
$$= x_1x_2 - y_1y_2 - i(x_1y_2 + x_2y_1) \quad ✓$$
$$\overline{z_1 z_2} = \overline{(x_1 + iy_1)(x_2 + iy_2)}$$
$$= \overline{x_1x_2 + i^2 y_1y_2 + ix_1y_2 + ix_2y_1}$$
$$= \overline{x_1x_2 - y_1y_2 + i(x_1y_2 + x_2y_1)}$$
$$= x_1x_2 - y_1y_2 - i(x_1y_2 + x_2y_1) \quad ✓$$
$$= \overline{z_1} \times \overline{z_2}$$

(3)

(ii) $z = x + iy \quad x = ? \quad y = ?$

$z(1+i) + \bar{z}(1-i) + 2z = 10 - 2i$

$(x+iy)(1+i) + (x-iy)(1-i) + 2x + 2iy = 10 - 2i$ ✓

$(x-y) + i(x+y) + (x-y) + i(-x-y) + 2x + 2iy = 10 - 2i$

$(x-y+x-y+2x) + i(x+y-y-x+2y) = 10 - 2i$

$(4x-2y) + i(2y) = 10 - 2i$

$Im: 2y = -2$

$y = -1$

$Re: 4x - 2y = 10$

$4x + 2 = 10$

$x = 2, \quad y = -1$

✓ ✓

(3)

12. (6 marks)

(a) $\boldsymbol{a}(t) = -9.8\,\boldsymbol{j}$

$\boldsymbol{v}(t) = \int -9.8\,\boldsymbol{j}\,dt = -9.8t\,\boldsymbol{j} + \boldsymbol{c}_1$ ✓

$\boldsymbol{v}(0) = 20cos(60^0)\boldsymbol{i} + 20sin(60^0)\boldsymbol{j} = 10\boldsymbol{i} + 10\sqrt{3}\,\boldsymbol{j} \Rightarrow \boldsymbol{c}_1 = 10\boldsymbol{i} + 10\sqrt{3}\,\boldsymbol{j}$

$\boldsymbol{v}(t) = 10\boldsymbol{i} + (10\sqrt{3} - 9.8t)\boldsymbol{j}$ ✓

$\boldsymbol{r}(t) = \int 10\boldsymbol{i} + (10\sqrt{3} - 9.8t)\boldsymbol{j}\,dt = 10t\,\boldsymbol{i} + (10\sqrt{3}t - 4.9t^2)\boldsymbol{j} + \boldsymbol{c}_2$

$\boldsymbol{r}(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \boldsymbol{c}_2 = \boldsymbol{j}$

$\boldsymbol{r}(t) = 10t\,\boldsymbol{i} + (10\sqrt{3}t - 4.9t^2 + 1)\boldsymbol{j}$ ✓

*If* $10t = 50, \ t = 5$ *and* $h = 10\sqrt{3}t - 4.9t^2 + 1$ ✓

*At* $t = 5, \ h = -34.9\ m$

This means the ball is not in flight for five seconds so Tom could not have kicked the ball through the window. ✓

(4)

(b) $h = 10\sqrt{3}t - 4.9t^2 + 1$

*At* $t = 3, \ h = 8.9\,m$

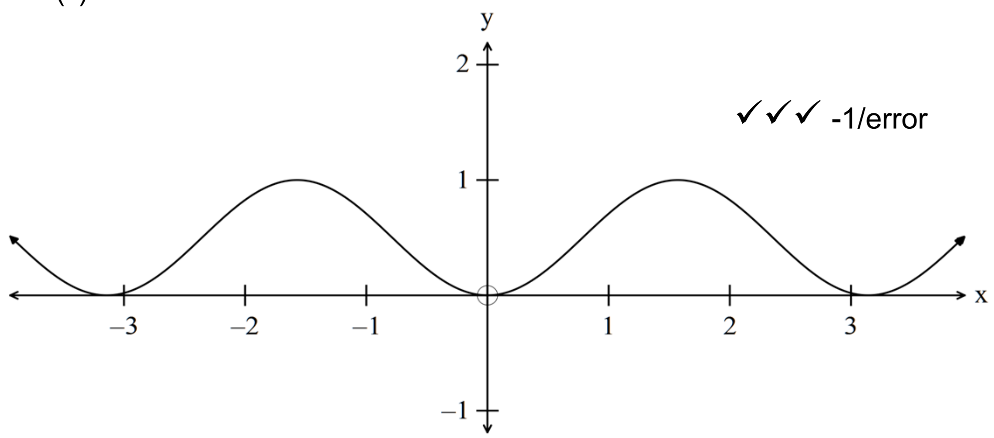
The ball was still in flight so the deputy may have seen it. ✓

(1)

13. (14 marks)

(a) (i) $y = g(f(x)) = g(sin(x)) = (sin(x))^2$ ✓ (1)

(ii)

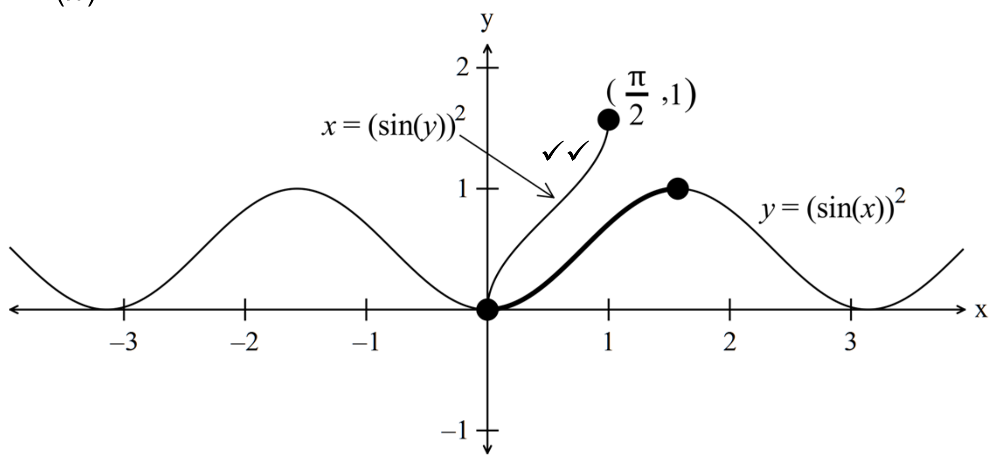



✓✓✓ -1/error

(3)

(iii) $a = \frac{\pi}{2}$ ✓✓ (2)

(iv)




(2)

(b) (i) $h(x) = \sqrt{x}$

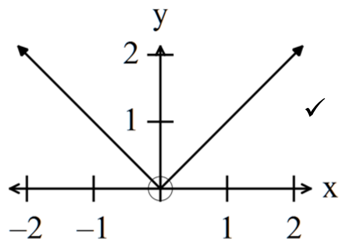
$h^{-1}(x) = x^2$ for $x \geq 0$ ✓ *must have restricted domain* (1)

(ii) $h(x) \geq 0$ ✓ (1)

(c) $g(h(x)) = g(\sqrt{x}) = x$ ✓

Defined on $[0, 2\pi]$ ✓ (2)

(d) (i)




✓

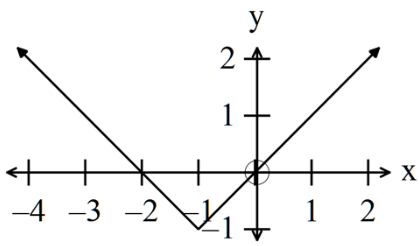
(1)

(ii) $y = |x|$ has no inverse because it is not a one to one function

i.e. for every $x$ there is not a unique $y$. ✓ (1)

14. (7 marks)

(a)




✓

$y = |x+1| - 1$

so $x < -2$ or $x > 0$ ✓ (2)

(b) Solve $|1+x| - 1 = |x-1|$

$y = |1+x| - 1 = \begin{cases} x & \text{for } x \geq -1 \\ -x-2 & \text{for } x < -1 \end{cases}$ ✓

$y = |x-1| = \begin{cases} x-1 & \text{for } x \geq 1 \\ 1-x & \text{for } x < 1 \end{cases}$ ✓

For $x \geq 1$

$x = x - 1$ No solution ✓

For $-1 < x < 1$

$x = 1 - x$

$2x = 1$

$x = \frac{1}{2}$ ✓

For $x < -1$

$-x - 2 = -x + 1$

$-2 = 1$ No solution ✓

$\therefore$ $x = \frac{1}{2}$ only

(5)

15. (4 marks)

✓✓

$$y = \frac{x^2}{(x-1)(x+3)}$$

✓ ✓

16. (5 marks)

(i)

$(1-i) = 1-i \quad = 1-i$

$(1-i)^2 = -2i \quad = -2i$

$(1-i)^3 = -2-2i \quad = -2(1+i)$

$(1-i)^4 = -4 \quad = -4$

$(1-i)^5 = -4+4i \quad = -4(1-i)$

$(1-i)^6 = 8i \quad = 4(2i)$

$(1-i)^7 = 8+8i \quad = 8(1+i)$

$(1-i)^8 = 16 \quad = 16$

$(1-i)^9 = 16-16i \quad = 16(1-i)$

$(1-i)^{10} = -32i \quad = -32i$ ✓✓ (2)

(ii) Every fourth result seems to be connected. ✓

Starting with $n=1$, then $n=5$, the pattern seems to be a multiple of $(1-i)$.
Starting with $n=2$, then $n=6$, the pattern seems to be a multiple of $i$.
Starting with $n=3$, then $n=7$, the pattern seems to be a multiple of $i$.
Starting with $n=4$, then $n=8$, the pattern seems to be a real multiple of 4.

✓ *for any one of these up to 3 marks*

The coefficients are a pattern of powers of 2. ✓

A lot more analysis is needed, but this is sufficient for 3 marks.

(3)

## END OF SECTION TWO